

# Rotations

Rotation:

A transformation that turns around a fixed point

Center of rotation:

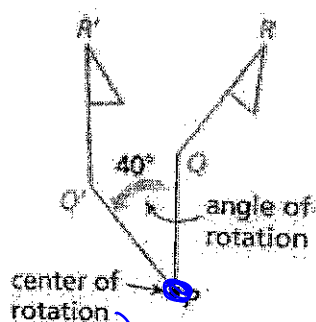
The point the picture rotates/turns around

Angle of rotation:

The number of degrees the picture rotates

- A rotation about a point  $P$  through an angle of  $x^\circ$  maps every point  $Q$  in the plane to a point  $Q'$  so that one of the following properties is true

- If  $Q$  is not the center of rotation  $P$ , then  $QP = Q'P$  and  $m\angle QPQ' = x^\circ$ , or
- If  $Q$  is the center of rotation  $P$ , then  $Q = Q'$ .



- Rotations can be clockwise or counterclockwise.  
All rotations are counterclockwise unless otherwise noted.

- We use a negative degree rotation to imply a clockwise rotation.

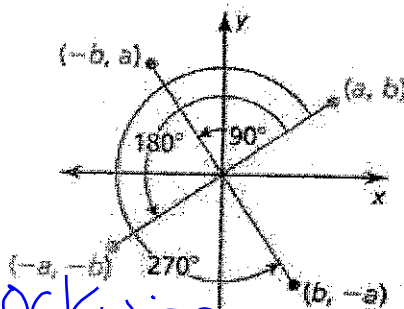
*(Negative Rotation)*

*(positive rotation)*

## Coordinate Rules for Rotations about the Origin

When a point  $(a, b)$  is rotated counterclockwise about the origin, the following are true:

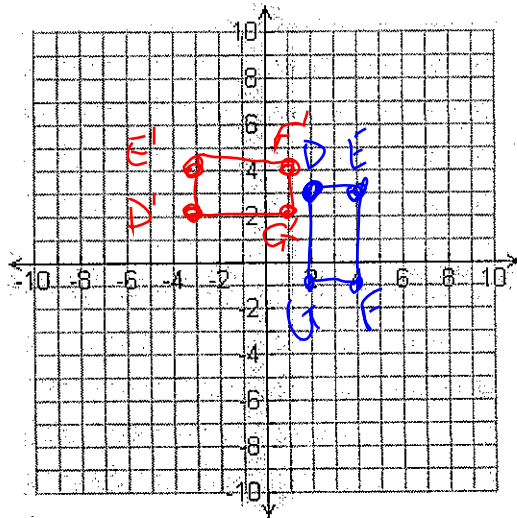
- For a rotation of  $90^\circ$ ,  $(a, b) \rightarrow (-b, a)$
- For a rotation of  $180^\circ$ ,  $(a, b) \rightarrow (-a, -b)$
- For a rotation of  $270^\circ$ ,  $(a, b) \rightarrow (b, -a)$
- For a rotation of  $360^\circ$ ,  $(a, b) \rightarrow (a, b)$



★ Always turn counterclockwise for Positive Rotations

Example 1:

Graph quadrilateral DEFG with vertices D(2, 3), E(4, 3), F(4, -1), and G(2, -1) and its image D'E'F'G' after a 90° rotation about the origin.



\*use the coordinate rule for 90° rotation about the origin\*

$R_{90}$  Turn/Switch order THEN change first sign  
 $(a, b) \rightarrow (-b, a)$

D (2, 3) → D' (-3, 2)

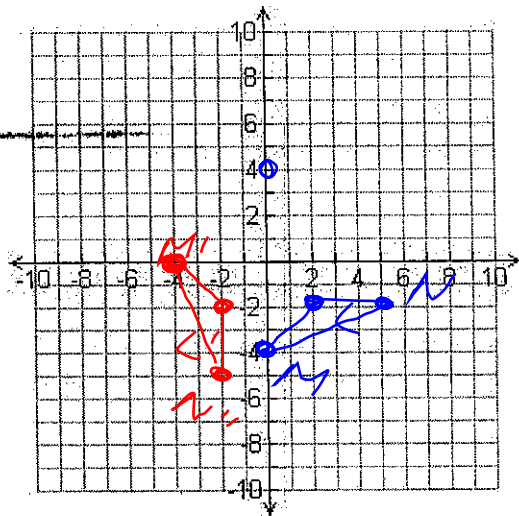
E (4, 3) → E' (-3, 4)

F (4, -1) → F' (+1, 4)

G (2, -1) → G' (+1, 2)

Example 2:

Graph triangle LMN with vertices L(2, -2), M(0, -4), and N(5, -2), and its image L'M'N' after a 270° rotation about the origin.



$R_{270}$  Turn/Switch order THEN change second sign

L (2, -2) → L' (-2, 2)

M (0, -4) → M' (-4, 0)

N (5, -2) → N' (-2, 5)

Practice Problems:

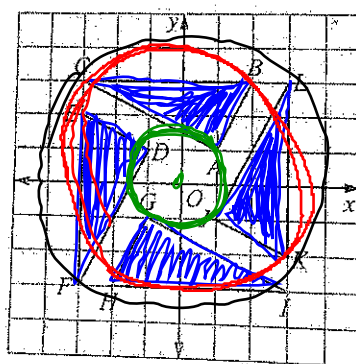
Use the coordinate rules to find the image of the the point after the following rotations.

1.  $R_{90}(2, 3) \rightarrow (-3, 2)$   
 $(4, 5) R_{270}(-5, 4) \rightarrow (-4, 5)$   
 $R_{180}(0, -1) \rightarrow (0, +1)$   
 $R_{90}(-4, -3) \rightarrow (3, -4)$

2.  $R_{180}(-1, 5) \rightarrow (1, -5)$   
 $R_{270}(-2, 3) \rightarrow (3, +2)$   
 $R_{270}(2, -2) \rightarrow (-2, -2)$   
 $R_{180}(-3, 1) \rightarrow (3, -1)$

In 3-16, the diagram shows that each triangle is the image of another triangle under a quarter turn about the origin. For example,  $R_{90}(\triangle ABC) = \triangle DEF$ .

+ counterclockwise  
 - clockwise



A → D → G → J  
 B → E → H → K  
 C → F → I → L

2.  $R_{90}(D) = ?$  G

5.  $R_{90}(GH) = ?$  JK

7.  $R_{180}(D) = ?$  J

9.  $R_{180}(KL) = ?$  EF

11.  $R_{270}(D) = ?$  A

13.  $R_{270}(\angle JKL) = ?$   $\angle GHI$

4.  $R_{90}(KL) = ?$  BC

6.  $R_{90}(\angle IGH) = ?$  ~~JKL~~  $\angle LJK$

8.  $R_{180}(GH) = ?$  AB

10.  $R_{180}(\angle DEF) = ?$   $\angle JKL$

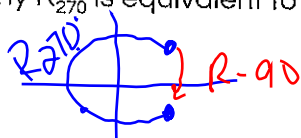
12.  $R_{270}(DF) = ?$  AC

14.  $R_{270}(\angle ACB) = ?$   $\angle JLK$

15. What transformation is equivalent to  $R_{180}$  followed by  $R_{90}$ ?

$180 + 90 = R_{270}$

16. Explain why  $R_{270}$  is equivalent to the clockwise rotation of  $R_{-90}$ ?



# Rotations

## Do Now:

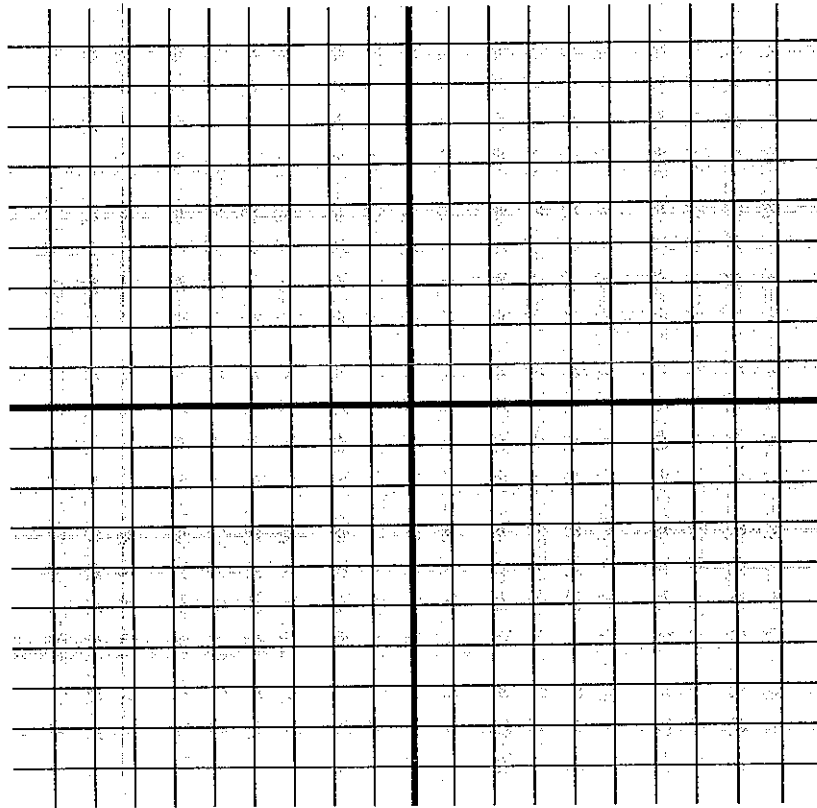
In the space provided, graph triangle ABC with points below, and its image A'B'C' after the following translation:

A(3, 2)

B(7, 4)

C(5, 8)

$R_{180}(\triangle ABC) \rightarrow$



## Finding the center of rotation

Center of rotation:

Steps for finding the center of rotation:

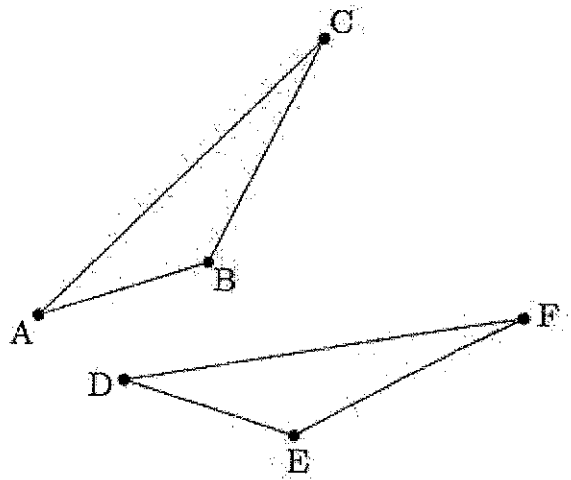
- 1) Using your straight edge, draw line segments connecting the corresponding points of the image and the preimage.
- 2) Construct the perpendicular bisector of the line segments.
- 3) Find the point where the perpendicular bisectors intersect.

Let's find the center of rotation for the Do Now.

Example 1:

In the accompany diagram, of preimage ABC and image DEF, find the center of rotation.

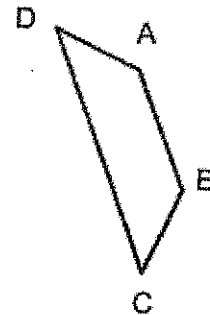
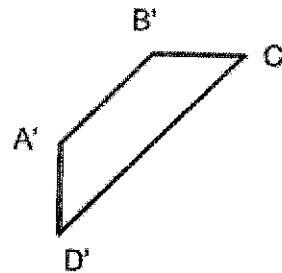
What is the approximate angle of rotation?



Example 2:

In the accompany diagram, of quadrilateral ABCD (preimage) and its image A'B'C'D', find the center of rotation.

What is the approximate angle of rotation?



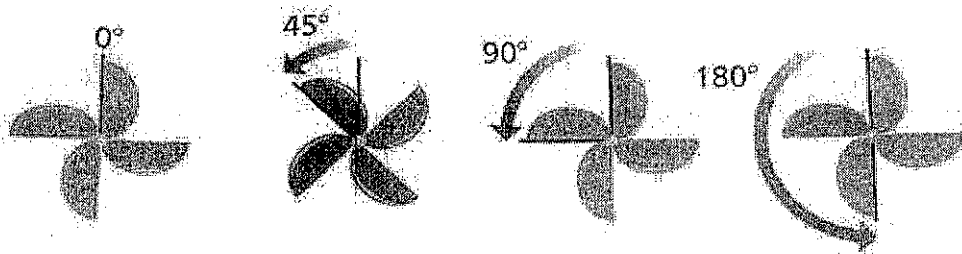
# Rotational Symmetry

Rotational symmetry:

the # of degrees it takes an object to map onto itself

Center of symmetry:

Angle of rotation:



- For a regular polygon, the smallest degree of rotational symmetry is  $360 \div n$ , where  $n$  is the number of sides.

Examples:

- Pentagon:

$$\frac{360}{5} = 72^\circ \text{ (or } 144, 216, 288, 360, \dots \text{)}$$

- Triangle:

$$\frac{360}{3} = 120^\circ \text{ (or } 240, 360, 480, \dots \text{)}$$

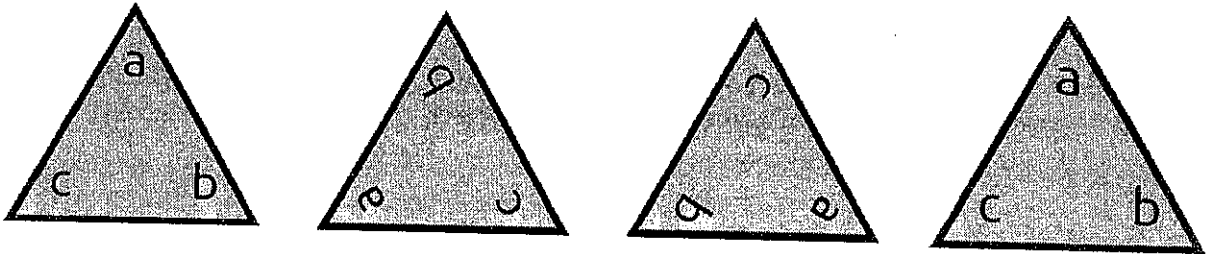
- Hexagon:

$$\frac{360}{6} = 60^\circ \text{ (or } 120, 180, 240, 300, \dots \text{)}$$

# Identity Symmetry

Identity symmetry:

Example:



$R_0$  ( $\triangle ABC$ )

$R_{120}$  ( $\triangle ABC$ )

$R_{240}$  ( $\triangle ABC$ )

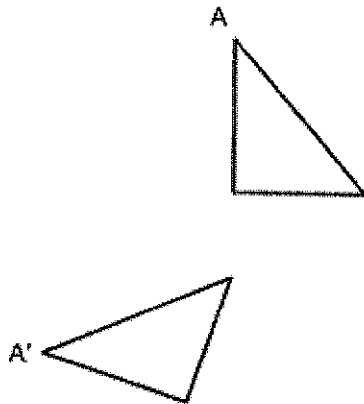
$R_{360}$  ( $\triangle ABC$ )

● Also referred to as the "\_\_\_\_\_ " symmetry.

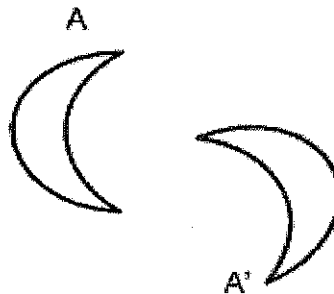
Practice Problems:

Construct the center of rotation for each of the figures below using a compass as a straight edge. Give an approximate angle of rotation.

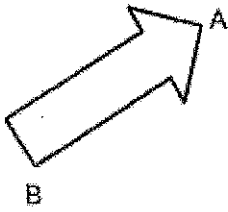
1.



2.



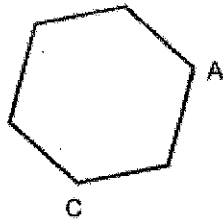
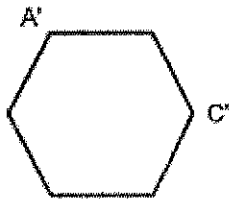
3.



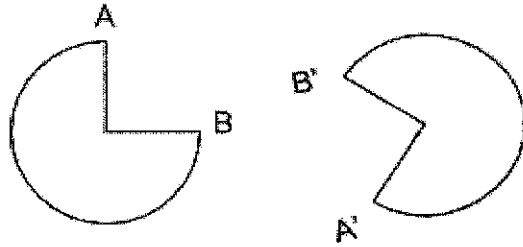
4.



5.

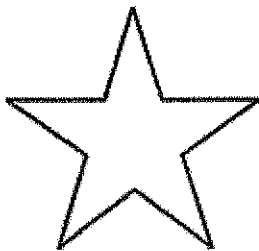


6.



State whether or not the figure has rotational symmetry. If so, find the angle of rotational symmetry and draw the center of rotation.

7.

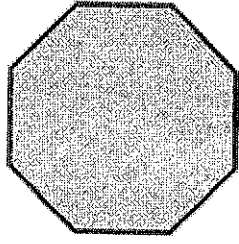


8.

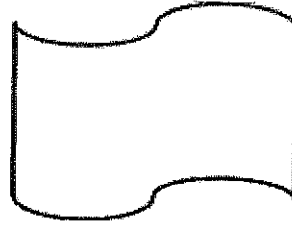




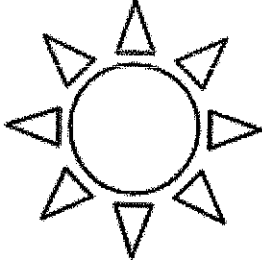
9.



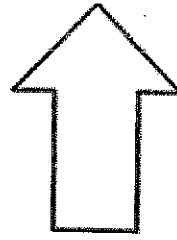
10.



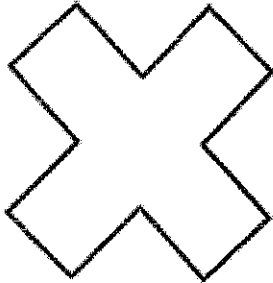
11.



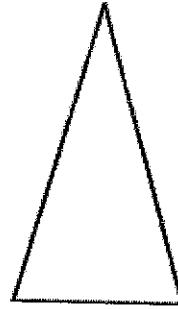
12.



13.



14.





Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Rotations

Rotation

R

R

Reflection

R

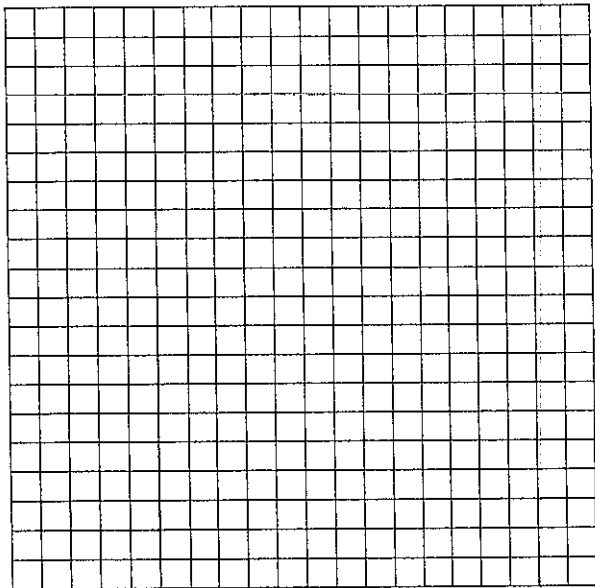
R

Translation

R

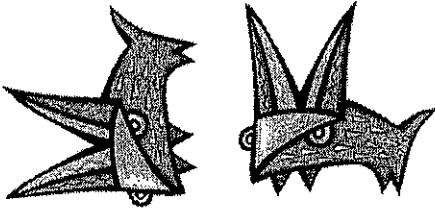
R

Rotation	Example	Rule
90 degrees counterclockwise		
180 degrees counterclockwise		
270 degrees counterclockwise		
360 degrees counterclockwise		
90 degrees clockwise		
180 degrees clockwise		
270 degrees clockwise		
360 degrees clockwise		

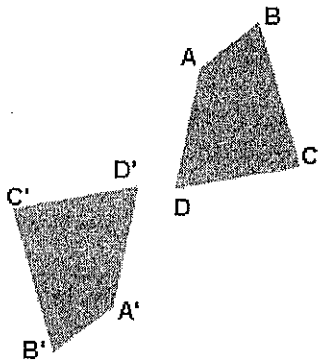


1. True or false

A. This picture has been rotated 180 degrees.



B. ABCD has been rotated 180 degrees.



C. Seahorse 1 has been rotated 90 degrees clockwise.



Seahorse 1



Seahorse 2

2.

If the letter **P** is rotated  $180^\circ$ , which is the resulting figure?

A) **q**

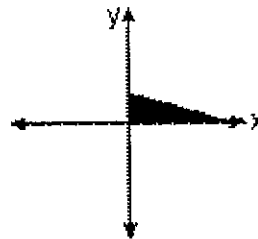
B) **b**

C) **a**

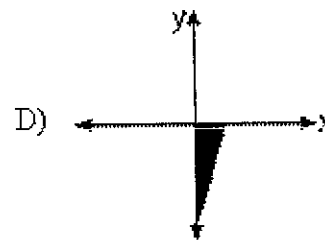
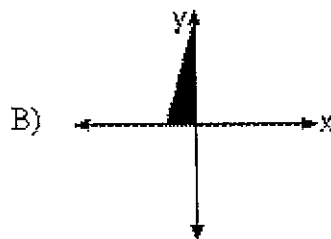
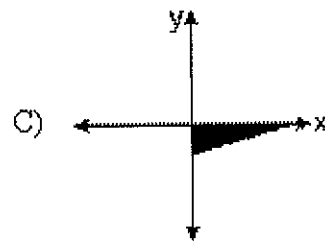
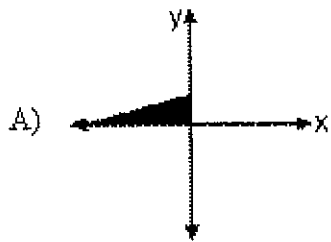
D) **d**

3.

The accompanying diagram shows a right triangle.



If the triangle is rotated  $90^\circ$  counterclockwise about the origin, what will the image look like?



4.

If the line with equation  $y = 2x + 3$  is rotated  $90^\circ$  counterclockwise about the origin, an equation of its image is

A)  $y = \frac{1}{2}x + \frac{3}{2}$

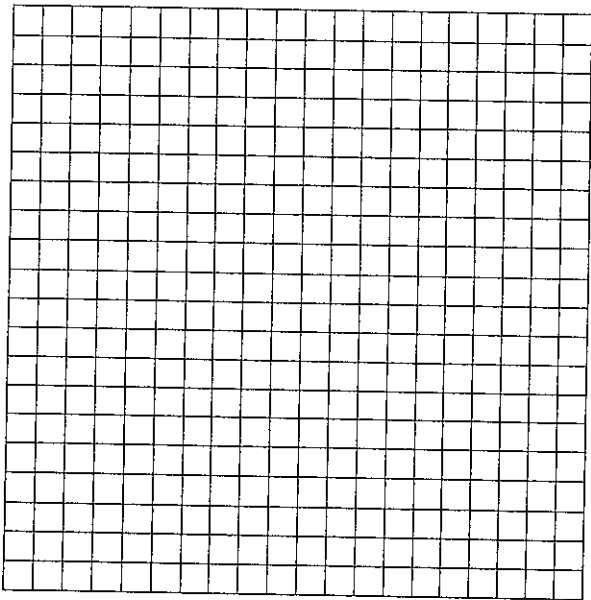
B)  $y = \frac{1}{2}x - \frac{3}{2}$

C)  $y = -\frac{1}{2}x - \frac{3}{2}$

D)  $y = -\frac{1}{2}x + \frac{3}{2}$

5. If  $A(-9, 8)$  is rotated about the origin  $270$  degrees, what are the coordinates of  $A'$ ?

6. The coordinates of triangle  $ABC$  are  $A(-1, 7)$ ,  $B(2, 0)$ , and  $C(0, -3)$ . Rotate triangle  $ABC$   $180$  degrees about the origin. Graph and state the coordinates of triangle  $A'B'C'$ .






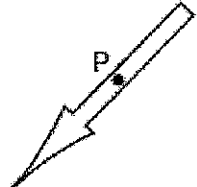
7. Point  $A$  is rotated  $180^\circ$  in a counterclockwise direction about the origin. If the coordinates of  $A$  are  $(-1, 3)$ , what are the coordinates of  $A'$ , its image?

8. What is the image of the point  $(2, -3)$  under a clockwise rotation of  $90^\circ$  ( $R_{-90^\circ}$ ) about the origin?

9. The accompanying diagram shows the starting position of the spinner on a board game.



How does this spinner appear after a  $270^\circ$  counterclockwise rotation about point  $P$ ?

- 1) 
- 2) 
- 3) 
- 4) 

10. A polygon is transformed according to the rule:  $(x, y) \rightarrow (x + 2, y)$ . Explain in detail what direction each point of the polygon moves.



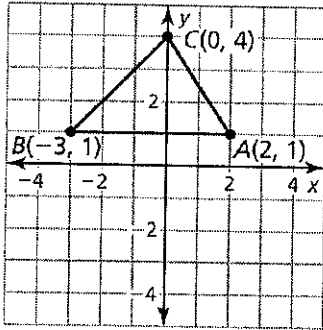
Name: \_\_\_\_\_

Date: \_\_\_\_\_

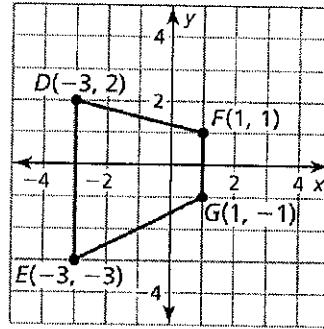
### Rotations & Review Homework

Graph the polygon's image after a rotation of the given number of degrees clockwise about the origin. State the new coordinates.

1.  $R 90^\circ$

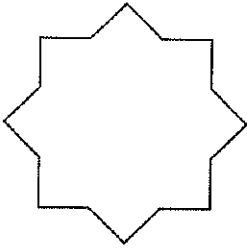


2.  $R 180^\circ$

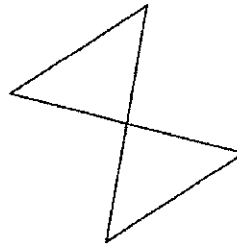


In Exercises 3 and 4, determine whether the figure has rotational symmetry. If so, describe any rotations that map the figure onto itself.

3.

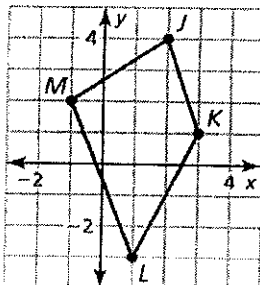


4.

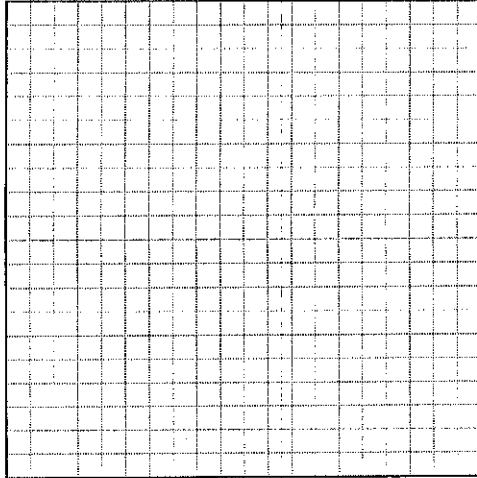


4. List the digits from 0-9 that have rotational symmetry.

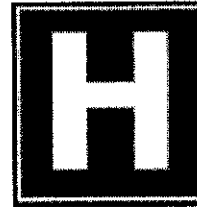
5. Graph and state the image of the given polygon after a  $270^\circ$  rotation about the origin.



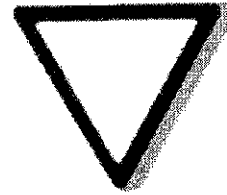
6. Draw  $\overline{AB}$  with points  $A(2, 0)$  and  $B(0, 2)$ . Rotate the segment  $90^\circ$  counterclockwise about point  $A$ . Then rotate the two segments  $180^\circ$  about the origin. What geometric figure did you create using the original segment and its images?



7. Does this sign have rotational symmetry?  
If so, what is the order of the symmetry?

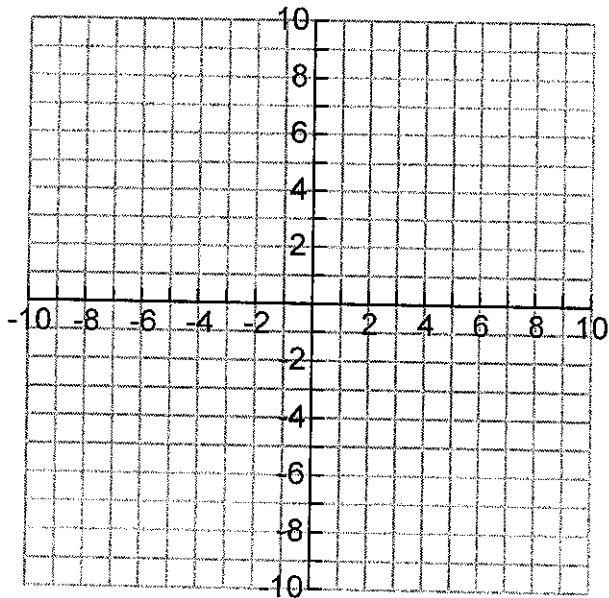


8. Does this sign have rotational symmetry?  
If so, what are the degrees of symmetry?



9. Which of the following letters or objects possesses rotational symmetry? Check ALL that apply.
- |                            |             |
|----------------------------|-------------|
| a. letter F                | e. a square |
| b. an equilateral triangle | f. letter W |
| c. letter X                | g. letter Q |
| d. a rectangle             | h. letter S |

Graph quadrilateral  $RSTU$  with vertices  $R(3, 1)$ ,  $S(5, 1)$ ,  $T(5, -3)$ , and  $U(2, -1)$  and its image after a  $270^\circ$  rotation about the origin.



### SOLUTION

Use the coordinate rule for a  $270^\circ$  rotation to find the coordinates of the vertices of the image. Then graph quadrilateral  $RSTU$  and its image.

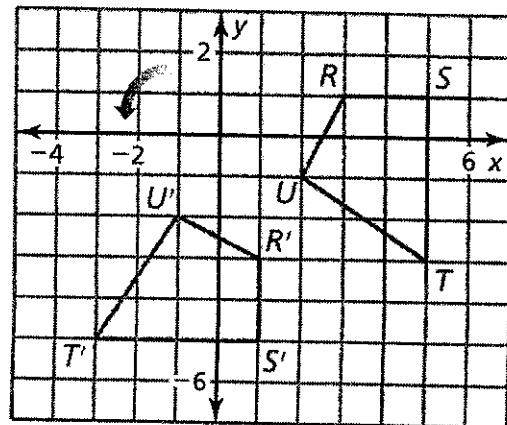
$$(a, b) \rightarrow (b, -a)$$

$$R(3, 1) \rightarrow R'(1, -3)$$

$$S(5, 1) \rightarrow S'(1, -5)$$

$$T(5, -3) \rightarrow T'(-3, -5)$$

$$U(2, -1) \rightarrow U'(-1, -2)$$



Graph  $\triangle ABC$  with vertices  $A(3, 1)$ ,  $B(3, 4)$ , and  $C(1, 1)$  and its image after a  $180^\circ$  rotation about the origin.

